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Inhomogeneous Cosmology with Spinning Fluid in High-Dimensional Space-Time

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We study the evolution of an inhomogeneous cosmology with spinning fluid in high-dimensional space-time. Using the Szekeres class II metric and the energy-momentum tensor derived by Ray and Smalley, we find evolving solutions, including an exponential inflation.

1. Ray and Smalley (1982) derived the energy-momentum tensor of a fluid with internal spin. Using the result, Som *et al.* (1988) investigated the evolution of an inhomogeneous and anisotropic cosmological model. Berman (1990) discussed inhomogeneous inflation with a spinning fluid. It goes without saying that gravitational effects of spin are significant in certain early stages of the evolution of the universe.

In this note we extend the results of Berman (1990) to the high-dimensional case and deal with the general situation of $\rho = Kp$. For different K, we find an evolving space-time and the relations between the evolution and the dimension D, the state parameter K, and the spin tensor S^{ab}.

2. We start with an extended Szekeres (1975) class II metric

$$ds^2 = -dt^2 + Q^2(t, x_1, x_a)\, dx_1^2 + R^2(t)(dx_2^2 + dx_3^2 + \cdots + dx_{D-1}^2) \qquad (1)$$

where Q, R are functions to be determined, D is the dimension of space-time, and the index $a = 2 \to D-1$.

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The Ricci tensors for the above metric are

$$R_{00}=(D-2)\ddot{R}/R+\ddot{Q}/Q$$
$$R_{11}=\sum_b QQ_{,bb}/R^2-(D-2)Q\dot{Q}\dot{R}/R-Q\ddot{Q}$$
$$R_{bb}=Q_{,bb}/Q-R\ddot{R}-(D-3)\dot{R}^2-R\dot{R}\dot{Q}/Q$$
$$R_{01}=0, \qquad R_{0b}=\dot{Q}_{,b}/Q-Q_{,b}\dot{R}/QR, \qquad R_{ab}=Q_{,ab}/Q$$

The energy-momentum tensor of the spinning fluid is (Som *et al.*, 1988)

$$T^{ij}=(\rho+S^{kh}\omega_{kh})u^iu^j+\left(p+\frac{1}{D-1}S^{kh}\omega_{kh}\right)(g^{ij}+u^iu^j)$$
$$+S_k^{(i}\omega^{j)k}+S_k^{(i}\sigma^{j)k}-\frac{1}{D-1}(g^{ij}S^{kh}\omega_{kh})+q^{(i}u^{j)} \tag{2}$$

where

$$q^i=S^{ik}\dot{u}_k+S^{ik}_{;k}-S^{kh}\omega_{kh}u^i, \qquad i,j,k,h=0\to D-1 \tag{3}$$

The S^{ij} are spinning tensors, and ω_{ij} is the angular velocity of the spin vector.

In comoving coordinate, $u^0=-u_0=1$, and the rest are $u_i=u^i=0$. We easily get

$$\sigma^{0i}=0, \qquad S^{ii}=0, \qquad \omega^{ij}=0 \qquad (i\neq j) \tag{4}$$

The nonnull components with spin in T^{ij} are only T^{0b} and $T^{0b}=S^{bc}{}_{;c}$. So Einstein's equations are

$$(D-2)\dot{Q}\dot{R}/QR+(D-2)(D-3)/2(\dot{R}/R)^2-\sum Q_{,bb}/QR^2=\rho \tag{5}$$

$$-(D-2)\ddot{R}/R-(D-2)(D-3)(\dot{R}/R)^2/2=p \tag{6}$$

$$-(D-3)\ddot{R}/R-(D-3)\dot{R}\dot{Q}/RQ-(D-3)(D-4)(\dot{R}/R)^2/2-\ddot{Q}/Q$$
$$+\sum_{c\neq b}Q_{,cc}/R^2Q=p \tag{7}$$

$$\dot{Q}_{,b}/Q-Q_{,b}\dot{R}/QR=R^2(QS^{bc})_{,c}/Q \tag{8}$$

$$Q_{,ab}=0 \tag{9}$$

From equation (9), we set

$$Q=f(t)\sum B_a(x_1)x_a \tag{10}$$

Then combining (6) with (7), we have

$$\ddot{R}/R-\ddot{f}/f+(D-3)[(\dot{R}/R)^2-\dot{R}\dot{f}/Rf]=0 \tag{11}$$

Evidently, $f=R$ is a solution of equation (11); another solution can be obtained from the following equations

$$(D-3)(\dot{R}/R)^2=\ddot{f}/f, \qquad \ddot{R}/\dot{R}=(D-3)\dot{f}/f \tag{12}$$

For $f=R$, considering the case of $\rho=Kp$, we get

$$\dot{Y}+(K+1)(D-1)Y^2/2K=0 \tag{13}$$

where $Y=\dot{R}/R$.

1. When $K=-1$, equation (13) has the solution

$$R=R_0\,e^{\lambda t} \tag{14}$$

It is an exponential inflationary solution.

2. When $K<-1$, then $0<\lambda=(D-1)(K+1)/2K<(D-1)/2$, and equation (13) has the solution

$$R\sim t^{1/\lambda} \tag{15}$$

It is a noninflationary solution.

3. When $K>0$, the solution is the same as (15), but $(D-1)/2<\lambda<\infty$.

4. When $0>K>-1$, we have

$$R\sim t^{-1/|\lambda|} \tag{16}$$

This solution shows $R\to 0$ as $t\to\infty$, and the evolving space disappears. So it has no physical significance.

Now we discuss another case; from (12) we get

$$R=f^{D-3} \tag{17}$$

To take $R=t^n$, we find

$$n=1-w^2/(w^2+w+1) \tag{18}$$

$$n=-1/(w-1) \tag{19}$$

where $w=D-3$.
Formula (18) gives the range of n as

$$0<n<\tfrac{2}{3} \tag{20}$$

and from (19) we have

$$-\infty<n<0 \tag{21}$$

Evidently $R=t^n$ is not an inflationary solution.

For spin the only nonnull components S^{bc} are given by

$$S^{bc}=h^{bc}(x_1)/Q \tag{22}$$

where $h^{bc}(x_1)$ are arbitrary functions of x_1.

According to the above discussion, we conclude that the evolution of space-time is closely related to the state parameter K. There is an inflationary solution when $K=-1$. Next the evolution is relative to the dimension of space-time, but there are no inflationary solutions when $K\neq-1$. The spin of the fluid affects only the metric Q; as can be seen, S^{bc} decreases exponentially or according to a power law with time.

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